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Ultrasonic characterization of porous absorbing materials: Inverse problem

Z.E.A. Fellah[a,*], F.G. Mitri[b], M. Fellah[c], E. Ogam[a], C. Depollier[d]

[a] *Laboratoire de Mécanique et d'Acoustique, CNRS-UPR 7051, 31 chemin Joseph Aiguier, Marseille 13009, France*
[b] *Mayo Clinic and Foundation, Department of Physiology and Biomedical Engineering, Ultrasound Research Laboratory, 200 First Street SW, Rochester, MN 55905, USA*
[c] *Laboratoire de Physique Théorique, Institut de Physique, USTHB, BP 32 El Alia, Bab Ezzouar 16111, Algeria*
[d] *Laboratoire d'Acoustique de l'Université du Maine, UMR-CNRS 6613, Université du Maine, Avenue Olivier Messiaen, 72085 Le Mans Cedex 09, France*

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Abstract

This paper concerns the ultrasonic characterization of air-saturated porous materials by solving the inverse problem using experimental data. It is generally easy to solve the inverse problem via transmitted waves, obtaining optimized values of tortuosity, viscous and thermal characteristic lengths, but this is not the case for the porosity because of its weak sensitivity in the transmitted mode. The reflection mode is an alternative to the transmission mode, in that it gives a good estimation of porosity and tortuosity by processing the data relative to measurements of the wave reflected by the first interface. The viscous and thermal characteristic lengths cannot be determined via the first interface reflection. The wave reflected by the second interface can be experimentally detected only for the weakly resistive porous materials. In this case, the characteristic lengths can be estimated. But for common air-saturated porous materials, the second reflection is very damped and its experimental detection is difficult. We propose in this paper to solve the inverse problem numerically by the least-squares method, using both reflected and transmitted experimental data. We determine simultaneously all the physical parameters intervening in the propagation. The minimization between experiment and theory is made in the time domain. The inverse problem is well posed, and its solution is unique. As with the classic ultrasonic approach for characterizing porous material saturated with one gas, the characteristic lengths are estimated by assuming a given ratio between them. Tests are performed using industrial plastic foams. Experimental and numerical results, and prospects are discussed.

1. Introduction

Ultrasonic characterization of porous materials saturated by air [1–6], such as plastic foams, fibrous or granular materials, is of a great interest for a wide range of industrial applications. The ultrasonic methods are not suitable for fibrous and granular media when the mean fibre diameter or grain dimension is of order of

*Corresponding author.
E-mail address: fellah@lma.cnrs-mrs.fr (Z.E.A. Fellah).

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1 mm or greater. These materials are frequently used in the automotive and aeronautics industries or in the building trade. The determination of the properties of a medium from waves that have been reflected by, or transmitted through, the medium is a classical inverse scattering problem [7–9]. Such problems are often approached by using a physical model of the scattering process, generating a synthetic response for some assumed values of the parameters, adjusting these parameters until reasonable agreement is obtained between the synthetic response and the observed data.

Ultrasonic characterization of materials is often achieved by measuring the attenuation coefficient and phase velocity in the frequency domain [5,6] or by solving the direct and inverse problems directly in the time domain [7–14]. The attractive feature of a time domain based approach is that the analysis is naturally limited by the finite duration of ultrasonic pressures and is consequently the most appropriate approach for the transient signal. However, for wave propagation generated by time harmonic incident waves and sources (monochromatic waves), the frequency analysis is more appropriate [1].

Acoustic damping in air-saturated porous materials is described by the inertial, viscous and thermal interactions between the fluid and the structure [1–4]. Depending on the temporal component of the acoustic excitation (pulse duration), the relaxation times describing the fluid–structure interactions are different. In the asymptotic domain [15] (corresponding to high-frequency range), the inertial, viscous and thermal interactions are modelled by causal integro-differential operators acting as a fractional derivatives [15]. These interactions are taken into account, by the tortuosity [1–4] for the inertial effects, and by the viscous characteristic length [16] and thermal characteristic length [1] for the viscous and thermal losses. In addition to these parameters, the porosity [17] is a key parameter playing an important role for all relaxation times. The determination of these parameters is crucial for the prediction of the sound damping in these materials.

At present, the experimental methods developed for solving such inverse problems (acoustic characterization) are based on measuring reflection [8,9] or transmission experimental data [5–7]. The reflected and transmitted mode are used separately. Each mode has its advantages and inconveniences for the characterization. The solution of the inverse problem via the experimental transmitted waves allow the determination of the tortuosity and the characteristic lengths. However, the porosity cannot be determined because of its low sensitivity in the transmitted mode [7]. The only alternative acoustic method for measuring the porosity is based on the solution of the inverse problem using the reflected waves by the first interface [8] of the porous slab. In this case, the determination of the porosity and tortuosity is possible just by measuring the wave reflected by the first interface. This wave has the advantage of being non-dispersive, and its experimental detection is easy.

For weakly resistive porous materials, the determination of the characteristic lengths is possible when the wave reflected by the second interface can be detected experimentally [9]. But in many cases, the waves reflected by the second interface are strongly attenuated, and their experimental detection is not easy. We can conclude that the use of transmitted or reflected modes separately are not sufficient for a complete characterization of the porous materials. This is the main reason why we propose in this paper to solve the inverse problem using both the reflected and transmitted experimental data (simultaneously). We try to take advantage of the two modes simultaneously for the experimental determination of all the parameters which intervene in the acoustics propagation in the asymptotic regime. Since it is not possible to solve the inverse problem for both the viscous and thermal characteristic lengths for porous material saturated by one fluid, they are estimated individually by assuming a given ratio between them.

The outline of the paper is as follows. Section 4 deals with a temporal equivalent fluid model; the connection between the fractional derivatives and the wave propagation in rigid porous media in the high-frequency range is established and the basic equations are written in the time domain. The expression of the reflection and transmission scattering operators are given. Section 3 is devoted to the inverse problem and the appropriate procedure, based on the least-squares method, which is used to estimate the acoustic parameters. Finally in Section 4, the experimental validation using ultrasonic measurement is discussed for air-saturated industrial plastic foams.

2. Temporal equivalent fluid model

The quantities involved in sound propagation in porous materials can be defined locally, on a microscopic scale. However, this study is generally difficult because of the complicated frame geometries. Only the mean values of the quantities involved are of practical interest. Averaging must be performed on a macroscopic scale, using volumes with large enough dimensions for the average to be significant. At the same time, these dimensions must be much smaller than the wavelength. Even on a macroscopic scale, describing sound propagation in porous material can be very complicated, since sound also propagates in the frame of the material. If the frame is motionless, the porous material can be replaced on a macroscopic scale by an equivalent fluid.

In porous material acoustics, a distinction can be made between two situations depending on whether the frame is moving or not. In the first case, the wave dynamics due to coupling between the solid frame and the fluid were clearly described in the Biot theory [18,19]. For the purposes of the present paper, the structure in air may be regarded as motionless and it can be assumed that waves propagate only in the fluid. This case is described by the equivalent fluid model which is a particular case of the Biot model, in which the acoustic field damping is described by the viscous and thermal exchanges between a fluid-saturated porous medium and its structure. In the asymptotic regime, corresponding to the high frequency limit [7,15], viscous and thermal interactions are modelled by the memory relaxation operators $\chi_v(t)$ and $\chi_{\text{th}}(t)$ given by [15]

$$\chi_v(t) = \frac{2\rho_f \alpha_\infty}{\Lambda}\sqrt{\frac{\eta}{\pi\rho_f}}\, t^{-1/2}, \tag{1}$$

$$\chi_{\text{th}}(t) = \frac{2(\gamma-1)}{K_a \Lambda'}\sqrt{\frac{\eta}{\pi Pr \rho_f}}\, t^{-1/2}, \tag{2}$$

where Pr is the Prandtl number, η and γ are the fluid viscosity and adiabatic constant, respectively. The model's relevant physical parameters are tortuosity α_∞, and viscous and thermal characteristic lengths Λ and Λ'. In this model the time convolution of $t^{-1/2}$ with a function, is interpreted as a semi-derivative operator according to the definition of the fractional derivative of order v given [20] by

$$D^v[x(t)] = \frac{1}{\Gamma(-v)}\int_0^t (t-u)^{-v-1} x(u)\,\mathrm{d}u, \tag{3}$$

where $\Gamma(x)$ is the Gamma function.

In this framework, the basic equations of our model can be written as:

$$\rho_f \alpha_\infty \partial_t \mathbf{v}(\mathbf{r},t) + \int_0^t \chi_v(t-t')\partial_t \mathbf{v}(\mathbf{r},t')\,\mathrm{d}t' = -\nabla p(\mathbf{r},t), \tag{4}$$

$$\frac{1}{K_a}\partial_t p(\mathbf{r},t) + \int_0^t \chi_{\text{th}}(t-t')\partial_t p(\mathbf{r},t')\,\mathrm{d}t' = -\nabla.\mathbf{v}(\mathbf{r},t). \tag{5}$$

Constitutive relations in the time domain result from arguments based on invariance under time translation and causality [13,14,21]. In these equations, p is the acoustic pressure, $\mathbf{v}$ is the particle velocity, ρ_f and K_a are the density and compressibility modulus of the fluid, respectively. The parameter α_∞ reflects the instantaneous response of the porous medium and describes the inertial coupling between the fluid and the structure. "Instantaneous" means that the response time is much shorter than the typical time scale for acoustic field variations. The susceptibility kernels χ_v and χ_{th} are memory functions which determine the dispersion of the medium.

We assume that the medium varies with depth x only and the incident wave is planar and normally incident. The acoustic pressure field is denoted by $p(x,t)$. It is assumed that the pressure field in the medium is zero prior to $t=0$. The wave equation for the acoustic pressure field of a porous dispersive medium with a rigid frame is obtained from the constitutive equations (4) and (5), and is of the form

$$\partial_x^2 p(x,t) - \frac{1}{c_0^2}\left[\alpha_\infty \partial_t^2 p(x,t) + \left(\alpha_\infty K_a \chi_{\text{th}} + \frac{\chi_v}{\rho_f} + c_0^2 \chi_{\text{th}} * \chi_v\right) * \partial_t^2 p(x,t)\right] = 0, \tag{6}$$

where $c_0=(K_a/\rho_f)^{1/2}$ is the speed of free fluid. The following notation is used for the convolution integral:

$$[f*g](x,t)=\int_0^t f(x,t-t')g(x,t')\,\mathrm{d}t'. \tag{7}$$

The propagation equation (6) can be written as

$$\frac{\partial^2 p(x,t)}{\partial x^2}-A\frac{\partial^2 p(x,t)}{\partial t^2}-B\int_0^t \frac{\partial^2 p(x,t)/\partial t'^2}{\sqrt{t-t'}}\,\mathrm{d}t'-C\frac{\partial p(x,t)}{\partial t}=0, \tag{8}$$

where coefficients A, B and C are constants given by

$$A=\frac{\rho_f\alpha_\infty}{K_a},\quad B=\frac{2\alpha_\infty}{K_a}\sqrt{\frac{\rho_f\eta}{\pi}}\left(\frac{1}{\Lambda}+\frac{\gamma-1}{\sqrt{Pr}\Lambda'}\right),\quad C=\frac{4\alpha_\infty(\gamma-1)\eta}{K_a\Lambda\Lambda'\sqrt{Pr}}, \tag{9}$$

respectively. The coefficient A concerns the velocity $c=1/\sqrt{\rho_f\alpha_\infty/K_a}$ of the wave in the air in the porous material. α_∞ has to do with the refractive index of the medium, which changes the wave velocity from $c_0=\sqrt{K_a/\rho_f}$ in free space to $c=c_0/\sqrt{\alpha_\infty}$ in the porous medium. The other coefficients essentially depend on the characteristic lengths Λ and Λ' and express fluid–structure viscous and thermal interactions. The constant B governs signal spreading while C is responsible for wave attenuation.

For a slab of porous material occupying the region $0\leqslant x\leqslant L$, the incident and scattered fields are related by the scattering operators (i.e., reflection and transmission operators) for the material. These are integral operators represented by

$$p^r(x,t)=\int_0^t \tilde{R}(\tau)p^i\left(t-\tau+\frac{x}{c_0}\right)\mathrm{d}\tau, \tag{10}$$

$$p^t(x,t)=\int_0^t \tilde{T}(\tau)p^i\left(t-\tau-\frac{L}{c}-\frac{(x-L)}{c_0}\right)\mathrm{d}\tau. \tag{11}$$

In Eqs. (10) and (11) the functions $\tilde{R}$ and $\tilde{T}$ are the reflection and the transmission kernels, respectively, for incidence from the left. These operators are independent of the incident field used in scattering experiment and depend only on the properties of the materials. The expressions of $\tilde{R}$ and $\tilde{T}$ are given by [7]:

$$\tilde{R}(t)=\frac{\sqrt{\alpha_\infty}-\phi}{\sqrt{\alpha_\infty}+\phi}\delta(t)-\frac{4\phi\sqrt{\alpha_\infty}(\sqrt{\alpha_\infty}-\phi)}{(\sqrt{\alpha_\infty}+\phi)^3}G\left(t,\frac{2L}{c}\right), \tag{12}$$

$$\tilde{T}(t)=\frac{4\phi\sqrt{\alpha_\infty}}{(\phi+\sqrt{\alpha_\infty})^2}G\left(t+\frac{L}{c},\frac{L}{c}\right), \tag{13}$$

where G is the Green's function [22] of the porous material:

$$G(t,k)=\begin{cases}0 & \text{if } 0\leqslant t\leqslant k,\\ \Xi(t)+\Delta\int_0^{t-k}h(t,\xi)\,\mathrm{d}\xi & \text{if } t\geqslant k,\end{cases} \tag{14}$$

with

$$\Xi(t)=\frac{b'}{4\sqrt{\pi}}\frac{k}{(t-k)^{3/2}}\exp\left(-\frac{b'^2k^2}{16(t-k)}\right), \tag{15}$$

where $h(\tau,\xi)$ has the following form:

$$h(\tau,\xi)=-\frac{1}{4\pi^{3/2}}\frac{k}{\sqrt{(\tau-\xi)^2-k^2}}\frac{1}{\xi^{3/2}}\int_{-1}^{1}\exp\left(-\frac{\chi(\mu,\tau,\xi)}{2}\right)(\chi(\mu,\tau,\xi)-1)\frac{\mu\,\mathrm{d}\mu}{\sqrt{1-\mu^2}} \tag{16}$$

and where $\chi(\mu,\tau,\xi)=(\Delta\mu\sqrt{(\tau-\xi)^2-k^2}+b'(\tau-\xi))^2/8\xi$, $b'=Bc_0^2\sqrt{\pi}$, $c'=C.c_0^2$ and $\Delta^2=b'^2-4c'$.

The first term on the right-hand side of Eq. (12): $((\sqrt{\alpha_\infty}-\phi)/(\sqrt{\alpha_\infty}+\phi))\delta(t)$ is equivalent to the instantaneous reflected response of the porous material. This term corresponds to the wave reflected by the first interface $x=0$. It depends only on the porosity and tortuosity of the material. The wave reflected by the first interface has the advantage of not being dispersive, but simply attenuated. This shows that it is possible to measure the porosity and tortuosity of the porous layer just by measuring the first reflected wave [7–9].

The second term on the right-hand side of Eq. (12): $-[(4\phi\sqrt{\alpha_\infty}(\sqrt{\alpha_\infty}-\phi))/(\sqrt{\alpha_\infty}+\phi)^3]G(t,2L/c)$ corresponds to reflection by the second interface at $x=L$. This term depends on the Green's function G of the medium which describes the propagation and dispersion of an acoustic wave making one round trip inside the porous slab. The Green's function depends on the tortuosity, viscous and thermal characteristics lengths Λ and Λ' of the material, but it does not depend on porosity. Experimentally, this second reflection contribution can be measured only for weakly resistive porous materials, because the acoustic signal is very attenuated.

Let us study the sensitivity of the porosity on the transmission scattering operator $\tilde{T}(t)$. By taking the derivative of $\tilde{T}(t)$ with respect to the porosity ϕ, we get

$$\frac{\partial\tilde{T}}{\partial\phi}=\frac{4\sqrt{\alpha_\infty}(\sqrt{\alpha_\infty}-\phi)}{(\sqrt{\alpha_\infty}+\phi)^3}G\left(t+\frac{L}{c},\frac{L}{c}\right),$$

so that when, $\phi\longrightarrow\sqrt{\alpha_\infty}$, the derivative $\partial\tilde{T}/\partial\phi\longrightarrow 0$. This means that when we tend to a free fluid, or for weakly resistive porous materials having a low value of porosity and tortuosity (near 1), the sensitivity of the porosity to the transmitted wave tends to zero. More generally, for a wide class of air-saturated porous materials, the term $\partial\tilde{T}/\partial\phi$ remains very small. Finally, we can conclude that the transmission scattering operator (Eq. (13)) depends on all the parameters, but the low sensitivity to the porosity makes impossible the retrieval of this parameter from transmitted experimental data.

In the next section, we solve the inverse problem using simultaneously the properties of both the reflection and transmission scattering operators Eqs. (12) and (13), and their dependence on the acoustical parameters.

3. Inverse problem

The basic inverse problem associated with the slab may be stated as follows: by measuring transmitted and/or reflected signals outside the slab, find the values of the medium's parameters ϕ, α_∞, Λ and Λ'. The inverse problem was solved in the transmission mode in Ref. [7] for α_∞, Λ and Λ', and gave a good correlation between experiment and theory for the transmitted signal. However, it has not been solved for the porosity because of its very weak sensitivity to this parameter in the transmitted mode.

The inverse problem was solved for the wave reflected by the first interface at normal [23] and oblique incidence [8], and gave good results for the tortuosity and porosity in plastic foams. Viscous and thermal characteristic lengths Λ and Λ' have been estimated for weakly resistive porous materials, when the experimental detection of the reflected wave by the second interface was possible [9]. The wave reflected by the first interface does not depend on Λ and Λ'. It is interesting to develop a method to measure all the parameters, using simultaneously the experimental reflected and transmitted data.

The solution of the direct problem involves a system of two operators (Eqs. (12) and (13)) expressed as functions of ϕ, α_∞, Λ and Λ'. The inversion algorithm for identifying the values of the slab parameters in the reflected and transmitted mode is based on the procedure: find the values of the parameters ϕ, α_∞, Λ and Λ' such that the reflected and transmitted signals describe the scattering problem in the best possible way (e.g., in the least-square sense). Solving the inverse problem for both the viscous and thermal characteristics lengths Λ and Λ' is very difficult, because these two parameters act simultaneously on dispersion (mathematically, we do not have a unique solution). In air-saturated porous materials the viscous and thermal interactions are both responsible of the dispersion but the viscous effects are more important [1]. As it has been used in the literature [24,25], the Λ'/Λ ratio is set to 3. At present, the only acoustical method able to measure the two parameters simultaneously is based on the principle of saturation the porous material by two fluids (air and Helium) [6]. At this time, and in our knowledge, it is not possible to measure the two characteristic lengths simultaneously by acoustic methods for porous material saturated by one fluid.

The inverse problem is to find values for parameters ϕ, α_∞ and Λ which minimize the cost function

$$U(\phi, \alpha_\infty, \Lambda) = \int_0^t (p^r_{\exp}(x,t) - p^r(x,t))^2\,\mathrm{d}t + \int_0^t (p^t_{\exp}(x,t) - p^t(x,t))^2\,\mathrm{d}t,$$

where $p^r_{\exp}(x,t)$ is the experimentally determined reflected signal, $p^r(x,t)$ the reflected wave predicted from Eq. (10), $p^t_{\exp}(x,t)$ the experimentally determined transmitted signal and $p^t(x,t)$ the simulated transmitted signal predicted from Eq. (11). However, because the equations are non-linear, the analytical method of solving the inverse problem using the conventional least-square method is tedious. In our case, a numerical solution of the least-square method can be found which minimizes the cost function $U(\phi, \alpha_\infty, \Lambda)$ defined by

$$U(\phi, \alpha_\infty, \Lambda) = \sum_{i=1}^{i=N} (p^r_{\exp}(x,t_i) - p^r(x,t_i))^2 + \sum_{i=1}^{i=N} (p^t_{\exp}(x,t_i) - p^t(x,t_i))^2, \tag{17}$$

where $p^r_{\exp}(x,t_i)_{i=1,2,\dots,N}$ is the discrete set of values of the experimental reflected signal, $p^r(x,t_i)_{i=1,2,\dots,N}$ is the discrete set of values of the simulated reflected signal, $p^t_{\exp}(x,t_i)_{i=1,2,\dots,N}$ is the discrete set of values of the experimental transmitted signal and $p^t(x,t_i)_{i=1,2,\dots,N}$ is the discrete set of values of the simulated transmitted signal. The next section discusses the solution of the inverse problem based on experimental reflected and transmitted data.

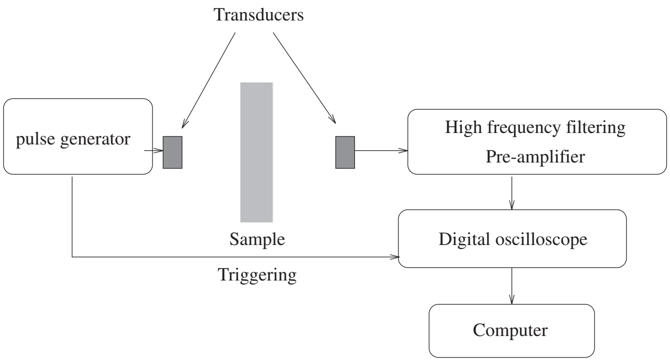

Fig. 1. Experimental set-up of the ultrasonic measurements in transmitted mode.

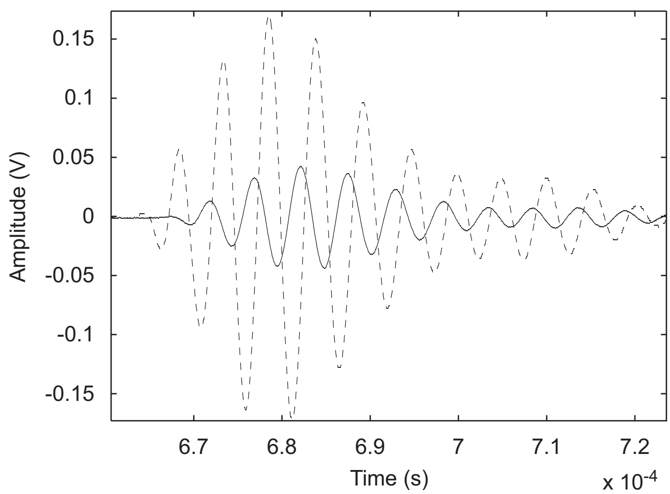

Fig. 2. Experimental incident signal (dashed line) and transmitted signal (solid line) using the pair of transducers Ultran NCG200-D13 (140–250 kHz).

4. Ultrasonic measurements

Experiments were performed in air using two pairs of broadband Ultran transducers, the first pair: NCG200-D13 with a central frequency of 195 kHz and a bandwidth of 6 dB extending from 140 to 250 kHz, and the second one: NCG100-D25 with a central frequency of 105 kHz and a bandwidth of 6 dB extending from 70 to 130 kHz.

Pulses of 400 V are provided by a 5058PR Panametrics pulser/receiver. The received signals are filtered above 1 MHz to avoid high-frequency noise. Electronic interference is eliminated by 1000 acquisition averages. The experimental setup is shown in Fig. 1. In the reflected mode, only one transducer of each pair is used simultaneously as a transmitter and receiver.

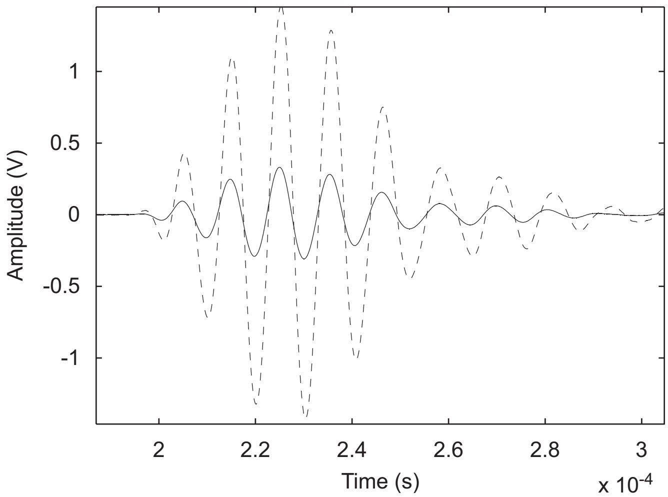

Fig. 3. Experimental incident signal (dashed line) and reflected signal (solid line) using the transducer Ultran NCG200-D13 (140–250 kHz).

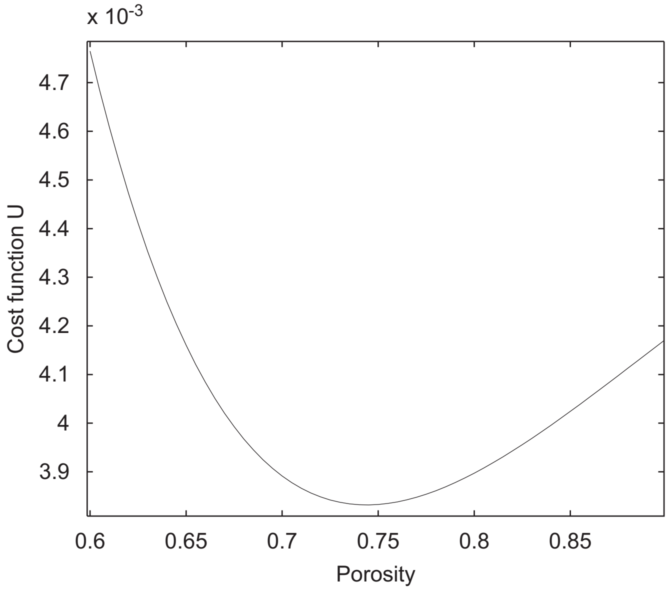

Fig. 4. Variation of the cost function U with porosity for $\alpha_\infty = 1.26$ and $\Lambda = 42\,\mu m$.

Consider a resistive sample of plastic foam, of thicknesses 0.7 ± 0.01 cm. Sample M1 was characterized using classic methods [5,7,8] and gave the following physical parameters $\phi = 0.7 \pm 0.05$, $\alpha_\infty = 1.25 \pm 0.05$, $\Lambda = (50 \pm 10)\,\mu$m.

First, consider the experimental data with the first pair of Ultran NCG200-D13 transducers. The measured signal generated by the transducer (measured without the sample) is given in Fig. 2 as a dashed line. The measured transmitted signal (obtained with the sample inserted) is given in the same figure (Fig. 2) as a solid line. The transmitted signal through the porous material is naturally attenuated, delayed and deformed due to the high dispersion at these frequencies.

In the reflection configuration (with one transducer acting as an emitter and receiver), we show in Fig. 3 the experimental reflected signal (solid line) and the experimental incident signal (dashed line). The incident signal

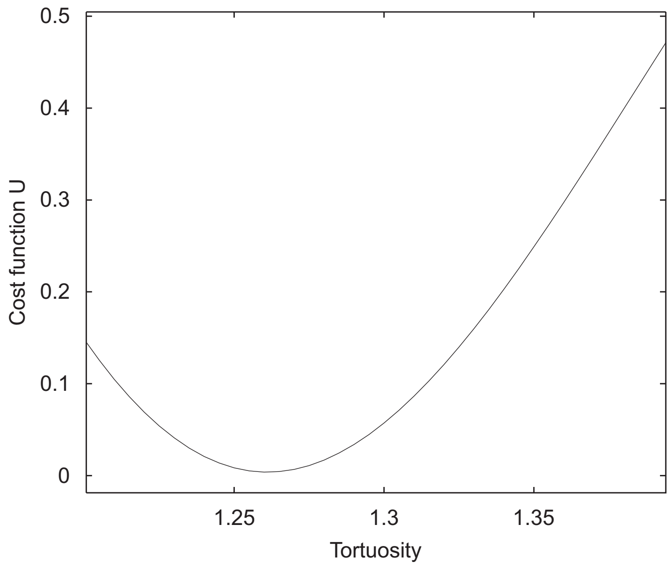

Fig. 5. Variation of the cost function U with tortuosity for $\phi = 0.74$ and $\Lambda = 42\,\mu$m.

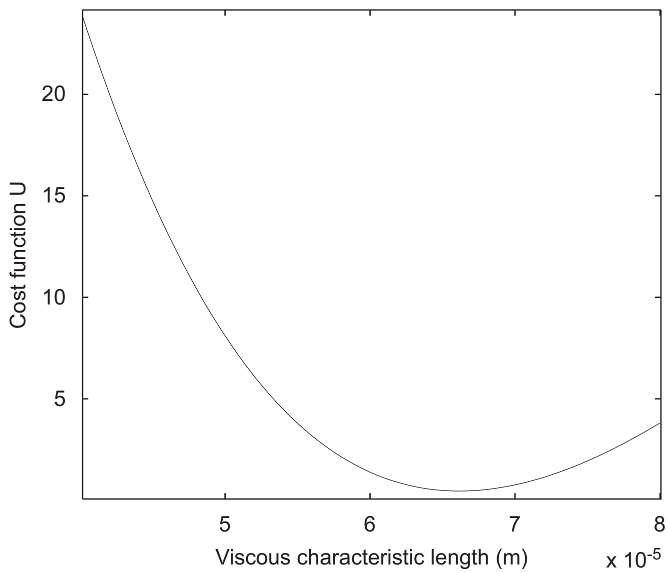

Fig. 6. Variation of the cost function U with viscous characteristic length for $\phi = 0.74$ and $\alpha_\infty = 1.26$.

was measured by putting an acoustic mirror (rigid plate) at the same position of the porous sample. The measured reflected signal corresponds to the first interface reflection (Eq. (12)); this is the reason why the two signals have the same arrival time.

By solving the inverse problem simultaneously using the experimental reflected and transmitted data (Eq. (17)), we find the following optimized values: $\phi = 0.74$, $\alpha_\infty = 1.26$ and $\Lambda = 66\,\mu$m. Using pair of these values, we present in Figs. 4–6, the variation of the cost function U given in Eq. (17) with porosity, tortuosity, and viscous characteristic length, respectively. The reader can see that the variation of the cost function with each physical parameter presents one minimum corresponding to the mathematical solution of the inverse problem. The fact that the cost function U exhibits a single

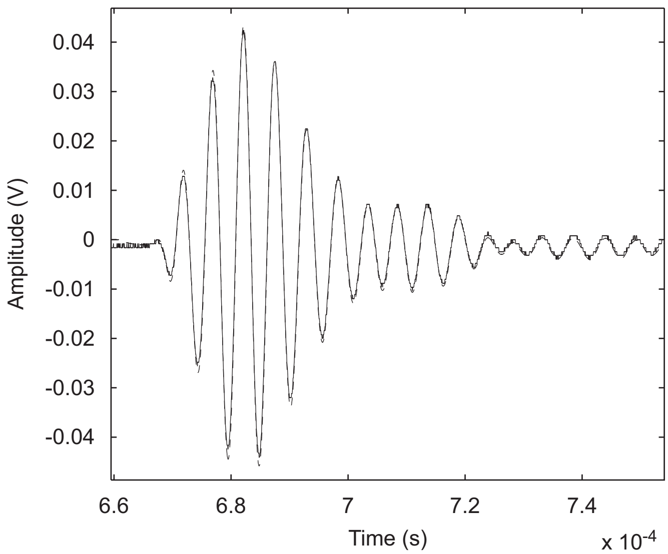

Fig. 7. Comparison between experimental transmitted signal (solid line) and simulated transmitted signal (dashed line): $\alpha_\infty = 1.26$, $\phi = 0.74$ and $\Lambda = 42\,\mu$m.

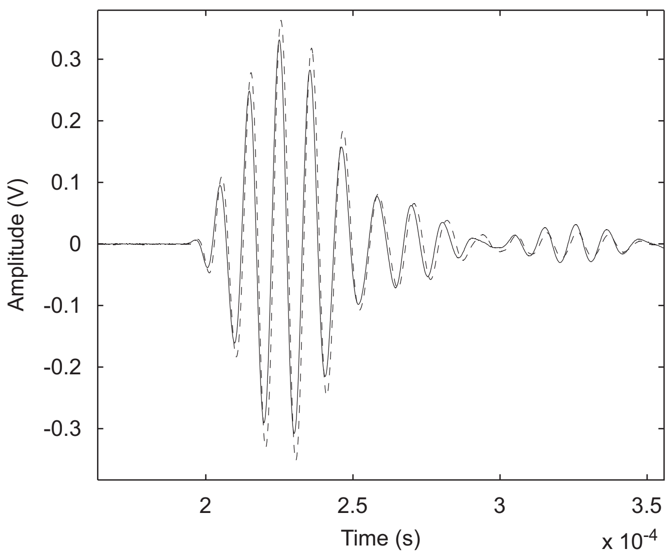

Fig. 8. Comparison between experimental reflected signal (solid line) and simulated reflected signal (dashed line): $\alpha_\infty = 1.26$, $\phi = 0.74$ and $\Lambda = 42\,\mu$m.

minimum shows that the inverse problem is well posed mathematically, and that there is uniqueness of the solution.

In Fig. 7, we compare the experimental transmitted signal and simulated transmitted signal for the optimized values of porosity, tortuosity and viscous characteristic length. A comparison between simulated and experimental reflected signals is given in Fig. 8. The difference between experiment and theory is slight for the reflected and transmitted modes, which leads us to conclude that the physical parameters are well-identified.

Let us now solve the inverse problem for the same porous sample using the second pair of transducers: Ultran NCG100-D25 with a central frequency of 105 kHz and a bandwidth of 6 dB extending from 70 to

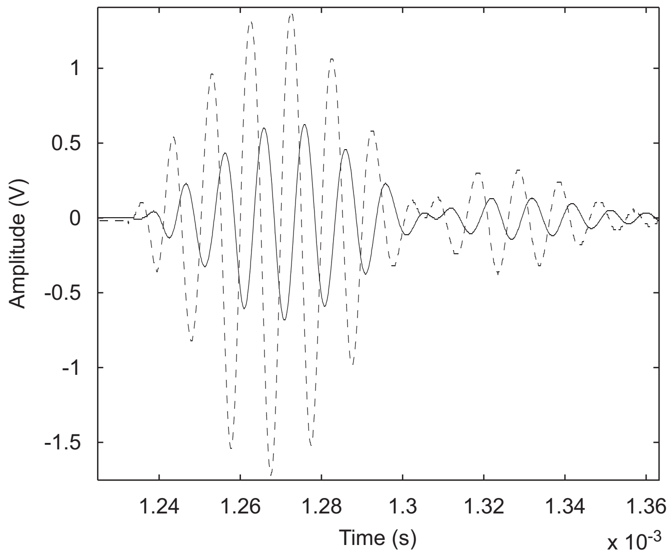

Fig. 9. Experimental incident signal (dashed line) and transmitted signal (solid line) using the pair of transducers Ultran NCG100-D25 (70–130 kHz).

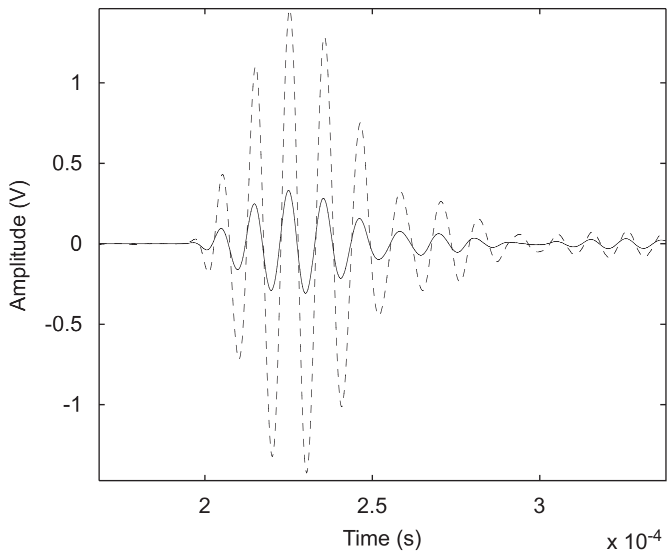

Fig. 10. Experimental incident signal (dashed line) and reflected signal (solid line) using the transducer Ultran NCG100-D25 (70–130 kHz).

130 kHz. The experimental incident and transmitted signals are given in Fig. 9. It can be seen in this case that the transmitted wave is less attenuated than in the previous case. This can be explained by the fact that the acoustic attenuation is less important in porous media for the lower frequencies [15]. Fig. 10 gives the experimental incident and reflected signal (using just one transducer for the measurement in reflected mode). The results, after solving the inverse problem using the reflected and transmitted experimental data are: $\phi = 0.72$, $\alpha_\infty = 1.21$, $\Lambda = 42\,\mu m$. The minima corresponding to these optimized values are presented in Figs. 11–13 for the porosity, tortuosity and viscous characteristic length. In Fig. 14, we compare the experimental transmitted signal and simulated transmitted signal for the optimized values of the parameters. A comparison between simulated and experimental reflected signals is given in Fig. 15. Here, again, we have

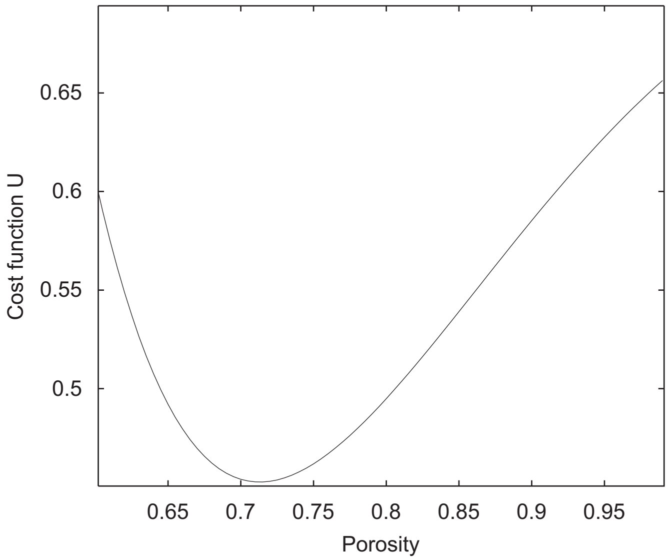

Fig. 11. Variation of the cost function U with porosity for $\alpha_\infty = 1.21$ and $\Lambda = 66\,\mu m$.

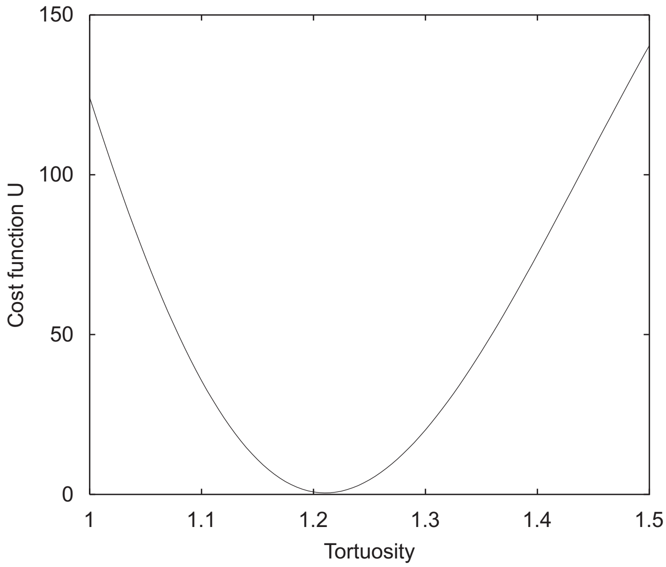

Fig. 12. Variation of the cost function U with tortuosity for $\phi = 0.72$ and $\Lambda = 66\,\mu m$.

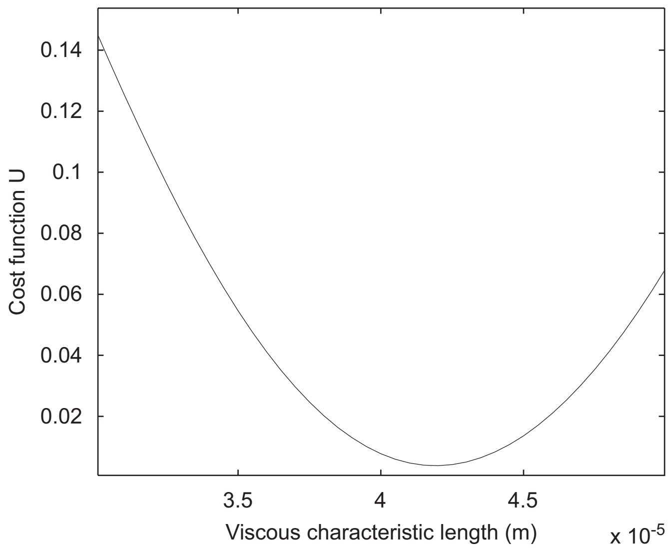

Fig. 13. Variation of the cost function U with viscous characteristic length for $\phi = 0.72$ and $\alpha_\infty = 1.21$.

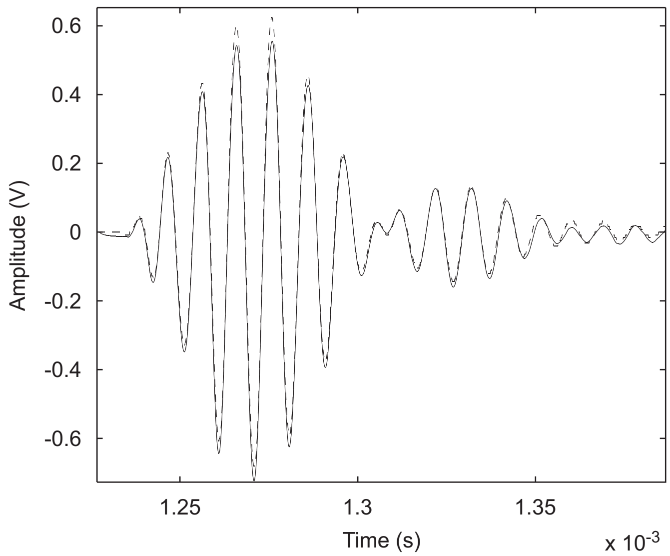

Fig. 14. Comparison between experimental transmitted signal (solid line) and simulated transmitted signal (dashed line): $\alpha_\infty = 1.21$, $\phi = 0.72$ and $\Lambda = 66\,\mu\text{m}$.

the uniqueness of the solution of the inverse problem with one clear minimum for each curve. In addition, the correlation of theoretical prediction and experimental data is good.

It can be seen that for the two pairs of transducers corresponding to two different frequency bands, the optimized values obtained using this method are close to those produced using classical methods [5,7,8]. This method seems to be efficient for estimating the physical parameters needed to describe sound propagation in air-saturated porous media such as plastic foams. The advantages of solving the inverse problem using both reflected and transmitted experimental data is the complete determination of all ultrasonic acoustic parameters (ϕ, α_∞ and Λ). By minimizing simultaneously on reflection and transmission, an average of the physical information contained in the two modes is obtained with a good precision.

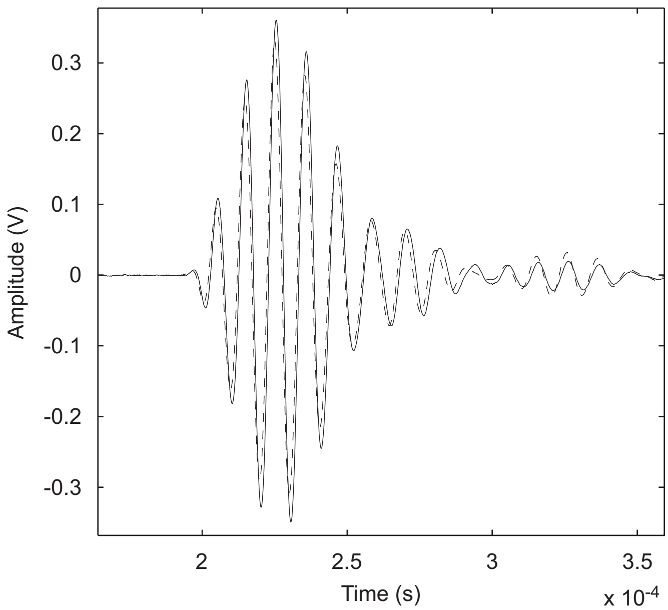

Fig. 15. Comparison between experimental reflected signal (solid line) and simulated reflected signal (dashed line): $\alpha_\infty = 1.21$, $\phi = 0.72$ and $\Lambda = 66\,\mu$m.

5. Conclusion

We have proposed in this paper a simple approach to solve the inverse problem using both reflected and transmitted waves, directly in the time domain employing the least-square method. Ultrasonic measurements in the transmission and reflection mode were processed using different transducers corresponding to the frequency bandwidths (70–130 kHz) and (140–250) kHz. A slight discrepancy was observed between theoretical predictions and experimental data in the two modes. In addition, the obtained parameters closely agree with those given by classical methods [5,7,8]. This leads to the conclusion that the proposed method is efficient. To make progress in this area, future projects will study the inverse problem using the experimental data at oblique incidence in reflection and transmission. In future we hope to solve the inverse problem and retrieve the physical parameters of porous material having an elastic frame, in which the dynamics of the structure must be taken into account (cancellous bone).

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